Homework: Big O

Name: _____

1. (5 points) Order the following functions by their growth rate: $n^4, n, \sqrt{n}, n^{1.5}, 12, n^3, n \log n, \frac{2}{n}, n \log n^2, 2^n, n^2$.

2. (5 points) Program x is $O(n)$. If it takes 10ms to run program x for $n = 100$, how long will it take to run for $n = 400$?

3. (5 points) Prove that $f(x) = 4x^2 - 5x + 3 = O(x^2)$.

4. (5 points) Prove that $f(x) = (x + 5) \log_2(3x^2 + 7) = O(x \log_2 x)$.

5. (5 points) Prove that $f(x) = \frac{x^2+5\log x}{2x+1} = O(x)$.

6. (5 points) Prove that $\frac{3x^4-2x}{5x-1} = O(x^3)$.

7. For each of the following functions, show the Big O of each using the simplest possible form. i.e. If the function is $O(n^2)$, do not answer $O(n^4)$.

   (a) (5 points) $n^2 + 3n$

   (b) (5 points) $3n^2 + 112n$

(c) (5 points) $64 + 128 + 256$

(d) (5 points) $218000 - 100n^3 + n^5$

(e) (5 points) $1 + n + n^2 + 5n^3 + \frac{1}{2}n^4$

(f) (5 points) $n^2 \times (n + 10) + n^2$

(g) (5 points) $\log_{10} 2^n + 12$

(h) (5 points) $1 + 2 + 3 + \ldots + n + n^2$

(i) (5 points) $n \times \frac{n-1}{2}$

8. For each of the following code snippets, provide an analysis of the running time in Big O notation. **Show your work!**

(a) (5 points)

```
sum = 0
for (i = 0; i < n; i++) loop
  sum++
end
```

(b) (5 points)

```
sum = 0
for (i = 0; i < n; i++) loop
  for (j = 0; j < n; j++) loop
    sum++
  end
end
```

(c) (5 points)

```
sum = 0
for(i = 0; i < sqrt(n)/2; i++) loop
  sum++
end

for(j = 0; j < sqrt(n)/4; j++) loop
  sum++
end
```

(d) (5 points)

```
sum = 0
for (i = 0; i < n; i++) loop
  for (j = 0; j < n * n; j++) loop
    sum++
  end
end
```

(e) (5 points)

```
i = 0
while (i < 128) loop
  input[i] = buffer[i]
  i++
end
```

(f) (5 points)

```
sum = 0
for (i = 0; i < n; i++) loop
  for (j = 0; j < m; j++) loop
    sum++
  end
end
```

(g) (5 points)

```
sum = 0
for (i = 0; i < n; i++) loop
  sum = sum + array[i]
  return sum
end
```

(h) (5 points)

```
sum = 0
for (i = 0; i < n/2; i++) loop
  sum = sum + array[i]
end
```

(i) (5 points)

```
sum = 0
for (i = 1; i < n/2; i = i * 2) loop
  sum = sum + array[i]
end
```

(j) (5 points)

```
odds = 0
evens = 0
for (i = 0; i < n; i = i++) loop
  if (i % 2 == 0) then
    evens++
  else
    odds++
  end
end
```

(k) (5 points)

```
sum = 0
m = n
for (i = 0; i < n; i++) loop
  for (j = 0; j < m; j++) loop
    sum++
  end
end
```

(l) (5 points)

```
sum = 0
for (i = 0; i < n; i = i + 5) loop
  for (j = 1; j < n; j = j * 3) loop
    sum++
  end
end
```